Tighter Spherical Harmonic Quantization Bound

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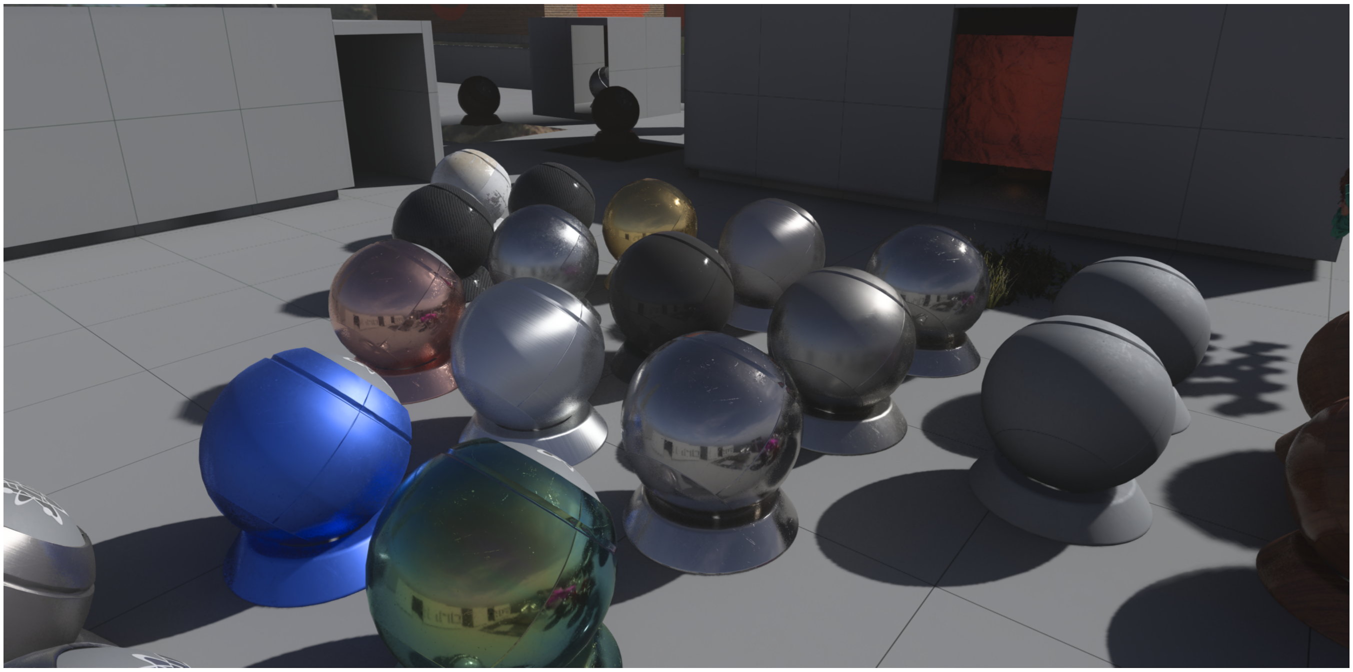

Figure 1: Test map PBR_White_Box, irradiance volume and models use Spherical Harmonics, statistics on the irradiance volume.

ABSTRACT

Spherical harmonics (SH) are a common function space used to encode lighting and other signals represented over the sphere. This article discusses a recent analytic bound to quantize the coefficients divided by the average value over the sphere, the first coefficient, of non-negative functions projected into this basis. The previous bound is derived, and while it is exact for the linear band, it turns out the non zonal quadratic functions have a tighter bound. This enables more efficient encoding of these signals.

KEYWORDS

spherical harmonics

ACM Reference Format:
Tyler Wiederien and Peter-Pike Sloan. 2018. Tighter Spherical Harmonic Quantization Bound. In *I3D Posters 2022*. ACM, New York, NY, USA, 2 pages. https://doi.org/XXXXXXX.XXXXXXX

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I3D '22, 2022, virtual, WA
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ACM ISBN 978-1-4503-XXXX-X/18/06…$0.00
https://doi.org/XXXXXXX.XXXXXXX

1 INTRODUCTION

Spherical harmonics are extensively used in interactive applications to represent lighting[1], visibility [2] and other signals on the sphere. A recent presentation [3] derived an analytic upper bound on the ratio of a SH projection coefficient to the average value over the sphere, DC. This enables SH to only use high precision (floating point) textures to encode DC, and 8bit formats to encode the higher bands divided by DC. The bound is independent of data but only applies when encoding the projection of non-negative functions, like lighting and visibility. It turns out that the non-zonal quadratic coefficients have a tighter analytic bound. This article derives the previous bound, and a slightly tighter bound for these quadratic functions.

2 BACKGROUND

2.1 Spherical Harmonics

The complex spherical harmonics are commonly used in non-graphics applications, we focus on the real spherical harmonics. Expressed in spherical coordinates:

$$Y_l^m \begin{cases} \sqrt{2}K_l^m \cos(m\phi)P_l^m(\cos\theta) & m > 0 \\ \sqrt{2}K_l^m \sin(|m|\phi)P_l^{|m|}(\cos\theta) & m < 0 \\ K_l^m P_l^m(\cos\theta) & m = 0 \end{cases} \tag{1}$$

where

$$K_l^m = \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} \tag{2}$$

and P_l^m are the associated Legendre polynomials.

2.2 Prior Bound

Following prior work [3] the projection of a function $f(x)$ into orthogonal basis $B_k(s)$ with Monte Carlo integration, leads to a sum of scaled delta functions:

$$b_j = \frac{1}{N}\sum_{i=1}^{N}\frac{B_j(x_i)f(x_i)}{p(x_i)} \tag{3}$$

The scale factor when importance sampling is $\frac{f(x_i)}{p(x_i)}$, and is non-negative when the function is non-negative. For SH, the DC basis function $Y_0^0(s)$ is non-negative, and independent of the location on the domain. When adding the delta functions, they simply sum, ie: only constructive interference occurs. For the linear basis, anti-podal points perfectly cancel each other out. Due to the fact that the basis is orthogonal any constant function results in a projection with perfect destructive interference for all bands - only DC is left.

SH are closed under rotation, so the Monte Carlo integral can be expressed as a sum of rotations of a canonical delta function. Choosing the Z axis for this canonical delta function, only the ZH functions are non-zero, the ratios in that frame can be used as an upper bound. The rotation of SH preserves the l^2-norm of the coefficients, so $\sum_{m=-l}^{l}(\frac{Y_l^m(x)}{Y_0^0})^2 = \frac{Y_l^0(z)}{Y_0^0} = 2l+1$, an individual coefficient has an upper bound that is $\sqrt{2l+1}$ and the sum of the squares of the coefficients has an upper bound that is $2l+1$.

The maximal value comes when projecting a delta function, not common when encoding indirect lighting. In that same presentation it is mentioned that taking the square root after $|x|$ is in $[0,1]$ and then squaring preserving the sign when reconstructing reduces the l^1 error by 50%, this is only used when not interpolating the coefficients due to the stronger non-linearity in reconstruction.

The coefficient bound is tight for the zonal functions, but the SH rotation matrix induced by a rotation in R^3 does not span the space of all rotation matrices in bands above linear, it is impossible to isolate all the energy in the non-zonal quadratic functions when projecting a non-negative function. This fact lead us to search for a tighter bound for the non-zonal quadratic coefficients.

3 NEW BOUND

3.1 Numerical Bound

Spherical harmonics were evaluated at 90301 positions on the unit sphere using a python script. The maximum absolute value for the non-zonal quadratic functions was 1.9365.

3.2 Analytic Bound

A maximum value after dividing by DC of $\frac{\sqrt{15}}{2}$ for Y_2^1 is found at $[\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{2}}, 0]$ in Cartesian coordinates, $(\frac{\pi}{2}, -\frac{\pi}{4})$ in (θ, ϕ). The derivative test from multi-variate calculus can be used to prove these are the local maxima of the basis functions.

3.3 Results

The new bound was tested on PBR_White_Box showing improvement between 2% and 9% as shown in table 1. Figure 2 shows this is due to the more even distribution of data after compression. A Shader Toy https://www.shadertoy.com/link with the compression and decompression code is online.

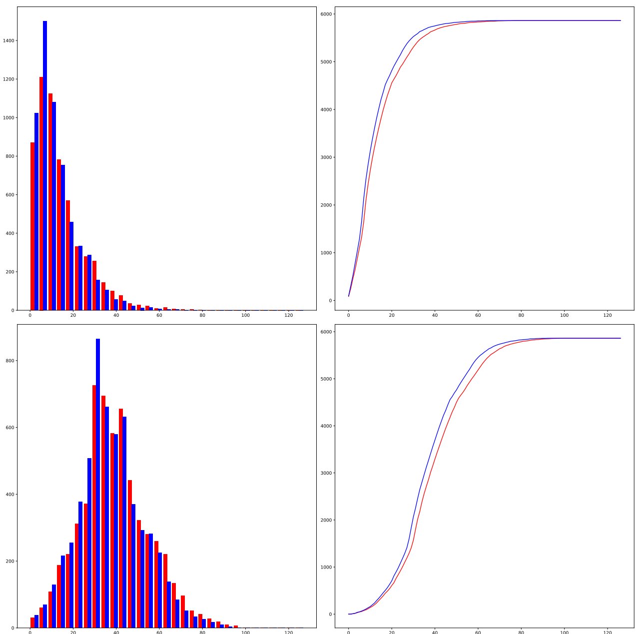

Figure 2: The red channel of the Y_2^{-2} from PBR_White_Box. Old bound is blue, new is red. Left is histogram of values, right is prefix sum.

Function	Old	New	Square Root Old	Square Root New
l^1 Error	1291.382	1232.067	720.100	704.648
l^2 Error	27.757	26.269	9.717	9.508
RMSE	0.067	0.066	0.040	0.039

Table 1: The errors after compression from PBR_White_Box

ACKNOWLEDGMENTS

Ari Silvennoinen pointed out that a tighter quadratic bound exists. Michal Iwanicki for feedback on an earlier draft.

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